Math 227A: Problem Set 1 and Further Suggested Exercises for Week 2

The following exercises comprise Problem Set 1, and are due on April 15:

1. Hatcher 4.1.10, 4.1.14, 4.2.8, 4.1.19.
2. Find an example of a continuous map $f : X \to Y$ between CW complexes for which each induced map $f_* : \pi_n(X, x_0) \to \pi_n(Y, y_0)$ is trivial but f is not a nulhomotopy.

The following are suggested exercises for Week 2:

1. Hatcher 4.1.12, 4.1.18, 4.2.9.
2. Adapt the proof we gave in class of the long exact sequence for a pair $A \subset X$ to a triple $B \subset A \subset X$:

   $$\cdots \to \pi_n(A, B, x_0) \to \pi_n(X, B, x_0) \to \pi_n(X, A, x_0) \to \pi_{n-1}(A, B, x_0) \to \cdots$$

   (This is proved in Hatcher, but you should make sure you can do it without referring to the text.)
3. The following exercise is optional and mostly targeted at people who like Heegaard Floer theory: Given a space X with the homotopy type of a CW complex, its nth *symmetric product* is the quotient of the product X^n by the action of the symmetric group S_n on the factors. More concretely, a point in $\mathrm{Sym}^n(X)$ is a g-tuple of unordered points in X.

   - What are the homotopy types of $\mathrm{Sym}^n(S^1)$ and $\mathrm{Sym}^n(\mathbb{C})$?
   - Let X be an n-punctured surface of genus g. What are the homotopy groups of $\mathrm{Sym}^{g+n-1}(X)$? (Hint: There is a relationship between symmetric products of wedge sums and symmetric products of ordinary product spaces.)
   - Restrict to the case that X is a sphere. Let $\{\alpha_1, \cdots, \alpha_n\}$ be a set of nonintersecting simple closed curves such that each component of $X - \{\alpha_1, \cdots, \alpha_n\}$ contains a single puncture. There is a torus $T_\alpha = \alpha_1 \times \cdots \times \alpha_n$ contained in $\mathrm{Sym}^{g+n-1}(X)$. Show (without explicitly writing down a map) that $\mathbb{T}_\alpha$ is a deformation retract of $\mathrm{Sym}^{n-1}(X)$.